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Tachyon vacuum in cubic superstring field theory

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Abstract: In this paper we give an exact analytic solution for tachyon condensation in the modified (picture 0) cubic superstring field theory. We prove the absence of cohomology and, crucially, reproduce the correct value for the D-brane tension. The solution is surprising for two reasons: First, the existence of a tachyon vacuum in this theory has not been definitively established in the level expansion. Second, the solution *vanishes* in the GSO(−) sector, implying a "tachyon vacuum" solution exists even for a *BPS* D-brane.

Keywords: Tachyon Condensation, String Field Theory, D-branes.

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1. Introduction

Since Schnabl provided [1] an analytic solution for tachyon condensation [2] in bosonic open string field theory, it has been of some interest to extend the analysis to superstrings. The most robust approach would utilize Berkovits's WZW-type superstring field theory [3], the only superstring framework for which reliable evidence for Sen's conjectures is available [4]. However, the nonpolynomial structure of the WZW action makes it a challenge to identify a solution and compute the brane tension analytically. Thus it seems worth reconsidering an old, if somewhat questionable proposal, which formulates superstring field theory as a cubic action for a picture number 0 string field — the so-called modified cubic superstring field theory [5] (see refs. [6, 7] for reviews). The action is,

$$S = \frac{1}{2}\langle\langle\Psi, Q_B\Psi\rangle\rangle + \frac{1}{3}\langle\langle\Psi, \Psi * \Psi\rangle\rangle \tag{1.1}$$

where Ψ is a ghost number 1, picture number 0 string field in the small Hilbert space of the matter+ghost superconformal field theory $X, \psi, b, c, \xi, \eta, \phi$. The only qualitative difference from the bosonic string is the definition of the bracket $\langle\langle,\rangle\rangle$, which requires insertions of two inverse picture changing operators at the open string midpoint. As a correlator in the upper half plane,

$$\langle\langle\Psi, \Phi\rangle\rangle = \langle Y_{-2}\, I \circ \psi(0)\, \phi(0)\rangle_{\text{UHP}} \qquad I(z) = -\frac{1}{z} \tag{1.2}$$

where (using the doubling trick),[1]

$$Y_{-2} = Y(i)Y(-i) \qquad Y(z) = -\partial\xi e^{-2\phi}c(z) \tag{1.3}$$

[1] There are actually many possible choices for Y_{-2}, defining inequivalent string field theories off-shell. We will stick with the definition eq. (1.3) since it the most canonical choice.

Level:	(0,0)	($\frac{1}{2}$,1)	(2,4)	(2,6)	($\frac{5}{2}$,5)
Percent Brane Tension	—	97%	108%	99%	91%

Table 1: Percent of brane tension produced for the tachyon vacuum in the modified cubic theory at various levels. Results taken from Raeymaekers [6] and Ohmori [9].

Since $Y(z)$ is dimension 0, BRST invariant and inserted at the midpoint, one can easily verify that all of the usual Chern-Simons like axioms are satisfied.

Though this action is very simple, as yet it is uncertain whether it defines an acceptable string field theory. One well-known objection [8] is that Y_{-2} has a nontrivial kernel, so the expected cubic equations of motion are reproduced only up to terms annihilated by Y_{-2}. However, the offending fields would be very singular at the string midpoint,[2] so it is unclear at what level this phenomenon will cause problems. Perhaps the ultimate test is to see whether the action reproduces the expected physics of tachyon condensation. Unfortunately, the answer is unclear [6, 10, 9]. A candidate vacuum solution has been identified at the first few levels, but — as can be seen in table 1 —the energy does not appear to converge. In fact, at level $(\frac{5}{2}, 5)$ an odd thing happens: the tachyon effective potential hits a singularity before the stable vacuum is reached, meaning that the (conjectured) nonperturbative vacuum lies on disconnected branch of the potential [6]. Despite these oddities, the striking thing about the energies in table 1 is that they are so close to the right answer. It is hard to believe this is a coincidence, but certainly more computations would be necessary to establish some sort of convergence.

In this paper we study the modified cubic theory from an analytic perspective, finding that — despite the above problems — the theory has a solution which can be interpreted as the endpoint of tachyon condensation. The crucial component is the calculation of the correct brane tension, which serves as a successful test of the cubic action. Surprisingly, the solution *vanishes* in the GSO($-$) sector, implying that the vacuum exists even for the field theory on a BPS brane.

This paper is organized as follows. In section 2 we give the algebraic setup and present the solution. We give a careful discussion of the "ψ_N piece" which requires some additional modification to reproduce the correct brane tension. In section 3 we evaluate the energy. Due to the extraordinary simplicity of the crucial correlator, the calculation is very easy — much simpler than for the bosonic string. In fact, we are even able to compute the energy directly in the $\mathcal{L}_0$ level expansion. We end with some conclusions.

2. Solution

We seek a generalization of Schnabl's solution for the cubic superstring equations of motion. The first step is identifying the relevant worldsheet degrees of freedom for expressing the

[2]The subalgebra of wedge states and related operators relevant for analytic calculations does not naturally produce fields in the kernel of Y_{-2}. On the other hand, it is difficult to study vacuum string field theory [9] with the canonical kinetic operator $\sim c(i)$ in this framework.

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solution. In split string notation [11 – 13], we claim these degrees of freedom are given by four string fields,

$$
\begin{aligned}
K &= \text{Grassmann even, gh\#} = 0 \\
B &= \text{Grassmann odd, gh\#} = -1 \\
c &= \text{Grassmann odd, gh\#} = 1 \\
\gamma^2 &= \text{Grassmann even, gh\#} = 2
\end{aligned} \tag{2.1}
$$

defined,

$$
\begin{aligned}
K &= -\frac{\pi}{2}(K_1)_L|I\rangle \qquad K_1 = L_1 + L_{-1} \\
B &= -\frac{\pi}{2}(B_1)_L|I\rangle \qquad B_1 = b_1 + b_{-1} \\
c &= -\frac{1}{\pi}c(1)|I\rangle \\
\gamma^2 &= \frac{1}{\pi}\gamma^2(1)|I\rangle \qquad \gamma^2(z) = \eta\partial\eta e^{2\phi}(z)
\end{aligned} \tag{2.2}
$$

where $|I\rangle$ is the identity string field and the subscript L denotes taking the left half of the corresponding charge (integrating the current counter-clockwise on the positive half of the unit circle). These fields satisfy the algebraic relations,

$$
\begin{aligned}
&[B, K] = 0 \qquad [B, \gamma^2] = 0 \qquad [c, \gamma^2] = 0 \\
&[B, c] = 1 \qquad B^2 = c^2 = 0
\end{aligned} \tag{2.3}
$$

and have BRST variations ($d = Q_B$),

$$
\begin{aligned}
dc &= cKc + \gamma^2 \qquad\qquad dB = K \\
d\gamma^2 &= cK\gamma^2 - \gamma^2 Kc \qquad dK = 0
\end{aligned} \tag{2.4}
$$

As another bit of notation, we denote

$$
F = e^{K/2} = \Omega^{1/2} \tag{2.5}
$$

for the square root of the $SL(2,\mathbb{R})$ vacuum $\Omega = e^K$.

With these preparations, the conjectured vacuum solution to the cubic equations of motion

$$
d\Psi + \Psi^2 = 0 \tag{2.6}
$$

is

$$
\Psi = Fc\frac{KB}{1-F^2}cF - FB\gamma^2 F \tag{2.7}
$$

We recognize the first term as Schnabl's solution for the bosonic string; the second term is a surprisingly simple superstring "correction." The solution is real and satisfies the Schnabl gauge $\mathcal{B}_0\Psi = 0$. There are many ways of "deriving" eq. (2.7), but perhaps the simplest is to translate Okawa's pure gauge form [11] using the modified BRST identities eq. (2.4).[3] Following Ellwood and Schnabl [18], the proof of absence of cohomology is immediate. We simply note the existence of a homotopy operator A satisfying,

$$d_\Psi A = dA + [\Psi, A] = 1 \tag{2.8}$$

The homotopy operator is the same as for the bosonic string,

$$A = -B\int_0^1 dt\Omega^t \tag{2.9}$$

since the B kills the correction term $-FB\gamma^2 F$ and the rest of the computation reduces to the bosonic derivation.[4]

Perhaps the most alarming aspect of the solution eq. (2.7) is the absence of GSO($-$) states. In particular, the solution exists even for the field theory on a *BPS* D-brane. While this is quite counterintuitive, we can offer some insight as to why this is possible, at least at the mathematical level. Note that, in some sense, the modified cubic theory has *two* tachyons: the physical tachyon in the GSO($-$) sector, corresponding to the vertex operator $\gamma(0)$; and the "auxiliary tachyon" in the GSO($+$) sector, corresponding to the vertex operator $c(0)$. The auxiliary tachyon does not represent a physical instability since $c(0)$ cannot be placed on shell. Nevertheless, the condensation of $c(0)$ is really what's responsible for the absence of open strings at the vacuum. One way of seeing this is through vacuum string field theory [20], which can be obtained from eq. (2.7) after an infinite reparameterization in the $\mathcal{L}_0$ level expansion. At the first two $\mathcal{L}_0$ levels the solution is,

$$\Psi = -FcF + \left(\frac{1}{2}FcKBcF - FB\gamma^2 F\right) + \ldots \tag{2.10}$$

Now perform an infinite reparameterization of the form discussed in refs. [12, 21],

$$\Psi \quad\to\quad \Psi_\alpha = \exp\left[\frac{1}{2}\ln\alpha(\mathcal{L}_0 - \mathcal{L}_0^*)\right]\Psi \tag{2.11}$$

with $\alpha \to 0$. To leading order, the solution becomes

$$\Psi_\alpha = -\frac{1}{\alpha}c + \ldots \mathcal{O}(\alpha^0) \tag{2.12}$$

and the corresponding kinetic operator is,

$$d_\Psi \quad\to\quad d_{\Psi_\alpha} = -\frac{1}{\pi\alpha}(c(1) - c(-1)) + \ldots \mathcal{O}(\alpha^0) \tag{2.13}$$

This is just the kinetic operator for (a form of[5]) vacuum string field theory. If the solution had some expectation value for the tachyon $\gamma(0)$, this would have appeared as a subleading

[3]Eq. (2.7) can also be derived from the marginal solution [14, 15] by setting $\lambda J = \lambda cKBc - \frac{\lambda}{1+\lambda}B\gamma^2$ and taking $\lambda \to \infty$, as suggested in ref. [16]. Actually this J is not "marginal" in the sense that it is not BRST invariant, but it does satisfy $d(\lambda J) + (\lambda J)^2 = 0$. Remarkably, this is sufficient for the full marginal solution to satisfy the equations of motion, as observed in ref. [17].

[4]For recent high-level studies of the spectrum around the tachyon vacuum, see ref. [19].

[5]This reparameterization squeezes towards the endpoints rather than the midpoint, which is why we obtain $c(1)$ rather than $c(i)$. The author thanks E. Fuchs for a useful discussion on this limit.

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divergence $\alpha^{-1/2}$ in the vacuum kinetic operator. Such terms can be accommodated into the vacuum string field theory framework [22], but really it is the leading divergence from the c ghost which is responsible for the absence of cohomology. Note that these comments naively apply to the Berkovits theory as well,[6] since there the role of Ψ is played by $e^{-\Phi}de^{\Phi}$. Thus we have the puzzling result that for superstrings, the tachyon is not necessary for describing physics around the tachyon vacuum.

2.1 ψ_N piece

To prove Sen's conjectures for the bosonic string it is necessary to regulate the solution and subtract a mysterious term — the "ψ_N piece" — which vanishes in the Fock space [1]. As we will see, a similar procedure is necessary for the superstring, but the story needs some refinement.

The necessity of the ψ_N piece can be understood from the requirement that the equations of motion hold in a sufficiently strong sense [11, 24]. It is straightforward to prove the equations of motion for eq. (2.7) using the identities eqs. (2.3), (2.4), but in the process we need to make the following assumption about the field $\frac{K}{1-F^2}$:

$$\frac{KF^2}{1-F^2} = \frac{K}{1-F^2} - K \tag{2.14}$$

This apparently innocuous equation is where the subtleties with the ψ_N piece come in.

The "obvious" solution to eq. (2.14) is to define $\frac{K}{1-F^2}$ as a geometric series expansion,

$$\frac{K}{1-F^2} = \lim_{N\to\infty} \sum_{n=0}^{N} K\Omega^n \tag{2.15}$$

Plugging in, one finds that eq. (2.14) is satisfied up to a term,

$$\lim_{N\to\infty} K\Omega^N \tag{2.16}$$

This actually vanishes in the Fock space as a power,

$$\frac{1}{N^3} \tag{2.17}$$

but for the bosonic string this is not rapid enough to ensure the equations of motion hold when contracted with the solution [11, 24]. For this purpose one needs eq. (2.14) to hold up to $1/N^4 \sim K^2\Omega^N$, which requires the sliver state to be subtracted from the geometric sum. This is the origin of the ψ_N piece.

For more general purposes it may be useful to have a definition where eq. (2.14) is satisfied up to an arbitrary inverse power of N in the Fock space. To see what the required corrections are, it is helpful to be more systematic. If we take eq. (2.14) as given, one finds upon recursive substitution the identity:

$$\frac{K}{1-F^2} = \sum_{n=0}^{N} K\Omega^n + \left(\frac{K}{1-F^2} - K\right)\Omega^N \tag{2.18}$$

[6] In fact, a GSO(+) vacuum solution to the Berkovits theory has already been conjectured in ref. [23]. The relation to our cubic solution is $e^{-\Phi}de^{\Phi} = \Psi$.

Now take the limit $N \to \infty$, and on the right hand side substitute the formal power series expansion,

$$\frac{K}{1-F^2} = -\sum_{n=0}^{\infty} \frac{B_n}{n!} K^n \tag{2.19}$$

in terms of Bernoulli numbers B_n. The result is the geometric expansion eq. (2.15) plus an infinite sequence of corrections involving powers of K acting on Ω^N. With a little Bernoulli arithmetic we can establish the following claim:

Claim: the expression,

$$\frac{K}{1-F^2} = \lim_{N\to\infty} \left[\sum_{n=0}^{N} K\Omega^n - \left(\sum_{k=0}^{A-2} \frac{B_k}{k!} K^k + K \right) \Omega^N \right] \tag{2.20}$$

is a solution to eq. (2.14) up to terms of order,

$$\lim_{N\to\infty} K^A \Omega^N \sim \frac{1}{N^{2+A}}$$

with $A \geq 1$.

Though at the moment it is not obvious, as it happens we will need $A = 3$ for the superstring. Therefore we will take ($B_1 = -\frac{1}{2}$),

$$\frac{K}{1-F^2} = \lim_{N\to\infty} \left[\sum_{n=0}^{N} K\Omega^n - \left(1 + \frac{1}{2} K \right) \Omega^N \right] \tag{2.21}$$

Plugging in to eq. (2.7) gives a regulated expression of the superstring solution,

$$\Psi = \lim_{N\to\infty} \left[\sum_{n=0}^{N} \psi'_n - \psi_N - \frac{1}{2}\psi'_N \right] - \Gamma \tag{2.22}$$

where,

$$\begin{aligned} \psi_n &= Fc\Omega^n BcF \\ \psi'_n &= \frac{d}{dn}\psi_n = Fc\Omega^n KBcF \\ \Gamma &= FB\gamma^2 F \end{aligned} \tag{2.23}$$

The correction $-\frac{1}{2}\psi'_N$ is new and was not necessary for the bosonic string.

3. Energy

Let us now calculate the energy. To prove Sen's conjecture we must demonstrate,

$$E = -S(\Psi) = -\frac{1}{2\pi^2} \tag{3.1}$$

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in the appropriate units.[7] Assuming the equations of motion and the regulated solution eq. (2.22), the energy breaks up into three terms:

$$E = -\frac{1}{6}\langle\langle\Psi, Q_B\Psi\rangle\rangle = E_B + E_\Gamma + E_{\psi'} \tag{3.2}$$

where E_B is the contribution from the "bosonic" part of the solution, E_Γ is the contribution from the superstring correction term, and $E_{\psi'}$ is the contribution from the additional "vanishing piece" $-\frac{1}{2}\psi'_N$. Explicitly,

$$\begin{aligned}
E_B &= -\frac{1}{6}\lim_{N\to\infty}\left[\sum_{m,n=0}^{N}\langle\langle\psi'_m, Q_B\psi'_n\rangle\rangle - 2\sum_{m=0}^{N}\langle\langle\psi'_m, Q_B\psi_N\rangle\rangle + \langle\langle\psi_N, Q_B\psi_N\rangle\rangle\right]\\
E_\Gamma &= -\frac{1}{6}\lim_{N\to\infty}\left[\langle\langle\Gamma, Q_B\Gamma\rangle\rangle - 2\sum_{m=0}^{N}\langle\langle\psi'_m, Q_B\Gamma\rangle\rangle + 2\langle\langle\psi_N, Q_B\Gamma\rangle\rangle\right] \qquad (3.3)\\
E_{\psi'} &= -\frac{1}{6}\lim_{N\to\infty}\left[\frac{1}{4}\langle\langle\psi'_N, Q_B\psi'_N\rangle\rangle - \sum_{m=0}^{N}\langle\langle\psi'_m, Q_B\psi'_N\rangle\rangle + \langle\langle\psi_N, Q_B\psi'_N\rangle\rangle + \langle\langle\psi'_N, Q_B\Gamma\rangle\rangle\right]
\end{aligned}$$

These expressions can be evaluated with knowledge of the inner products,

$$\langle\langle\psi_m, Q_B\psi_n\rangle\rangle \quad \langle\langle\psi_n, Q_B\Gamma\rangle\rangle \quad \langle\langle\Gamma, Q_B\Gamma\rangle\rangle \tag{3.4}$$

An elementary computation with eqs. (2.3), (2.4) reduces these to a correlator on the cylinder (see figure 1):

$$\langle\langle\Omega^x Bc\Omega^y c\Omega^z\gamma^2\rangle\rangle = \left\langle Y_{-2}\int_{i\infty}^{-i\infty}\frac{dw}{2\pi i}b(w)c(y+z)c(z)\gamma^2(0)\right\rangle_{C_{x+y+z}} \tag{3.5}$$

We evaluate this in appendix A, finding:

$$\langle\langle\Omega^x Bc\Omega^y c\Omega^z\gamma^2\rangle\rangle = \frac{x+y+z}{2\pi^2}y \tag{3.6}$$

The inner products become,

$$\begin{aligned}
\langle\langle\psi_m, Q_B\psi_n\rangle\rangle &= \frac{m+n+2}{\pi^2}\\
\langle\langle\psi_m, Q_B\Gamma\rangle\rangle &= \frac{1}{\pi^2}\\
\langle\langle\Gamma, Q_B\Gamma\rangle\rangle &= 0
\end{aligned} \tag{3.7}$$

This result is vastly simpler than for the bosonic string, where $\langle\psi_m, Q_B\psi_n\rangle$ is a rather unwieldy expression involving trigonometric functions [1].

[7] We normalize the basic correlator in the upper half plane,

$$\langle c\partial c\partial^2 c(z_1)e^{-2\phi}(z_2)\rangle_{\rm UHP} = -2$$

and set α', the open string coupling, and the spacetime volume factor to unity. Our normalization of the action agrees with Ohmori [9].

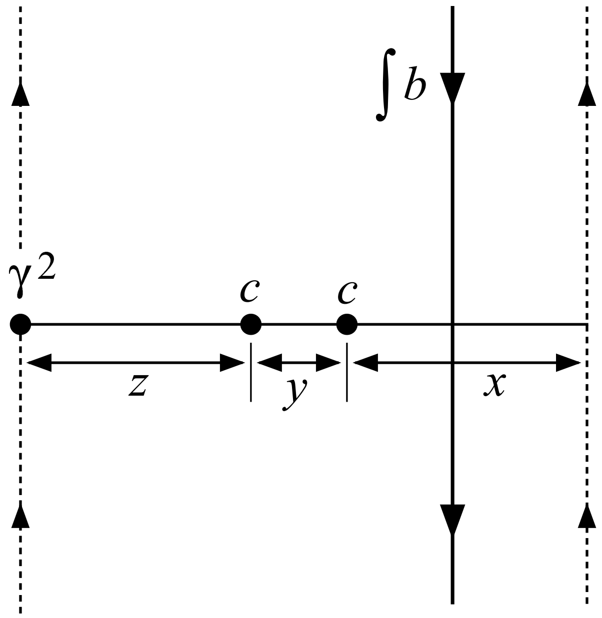

Figure 1: Correlator eq. (3.5) on the cylinder. The dashed vertical lines are identified, and the picture changing operator Y_{-2} is inserted at the midpoint, $\pm i\infty$ in this coordinate system.

It is now extremely straightforward to evaluate the energy. Calculating the "bosonic" contribution first,

$$\begin{aligned} E_B &= -\frac{1}{6}\lim_{N\to\infty}\left[0 - 2\sum_{n=0}^{N}\frac{1}{\pi^2} + \frac{2N+2}{\pi^2}\right] \\ &= -\frac{1}{6}\lim_{N\to\infty}\left[-\frac{2(N+1)}{\pi^2} + \frac{2(N+1)}{\pi^2}\right] \\ &= 0 \end{aligned} \tag{3.8}$$

We make a few comments. First, note that the "Fock space" contribution to the energy, from the double sum, vanishes identically because $\langle\langle\psi'_m, Q_B\psi'_n\rangle\rangle = 0$. This is consistent with the expectation that the pure gauge solutions of Schnabl [1] have vanishing energy. Second, note that,

$$\langle\langle\psi_N, Q_B\psi_N\rangle\rangle$$

diverges linearly for large N, though fortunately this divergence cancels out of E_B. Thus, the inner product $\langle\langle\psi'_N, Q_B\psi_N\rangle\rangle$ will *a priori* make a finite contribution to the energy, which is why the subleading correction $-\frac{1}{2}\psi'_N$ to the ψ_N piece is important.

The contributions from E_Γ and E'_ψ are easily seen to be,

$$\begin{aligned} E_\Gamma &= -\frac{1}{6}\lim_{N\to\infty}\left[0 - 0 + 2\cdot\frac{1}{\pi^2}\right] = -\frac{1}{3\pi^2} \\ E_{\psi'} &= -\frac{1}{6}\lim_{N\to\infty}\left[0 - 0 + \frac{1}{\pi^2} + 0\right] = -\frac{1}{6\pi^2} \end{aligned} \tag{3.9}$$

Adding everything up,

$$E = 0 - \frac{1}{3\pi^2} - \frac{1}{6\pi^2} = -\frac{1}{2\pi^2} \tag{3.10}$$

recovering the expected vacuum energy.

We can also prove that the equations of motion are satisfied when contracted with the solution. Though this essentially follows from our previous discussion in section 2.1, it is worthwhile to check. We need to evaluate the cubic term,

$$\langle\langle \Psi, \Psi * \Psi \rangle\rangle \tag{3.11}$$

Since the picture changing insertion Y_{-2} has ϕ-momentum -4 and we need ϕ-momentum -2 to get a nonvanishing correlator, the only nonvanishing contributions to the cubic term involve two ψ_ms and one Γ. Furthermore, because the correlator eq. (3.6) is linear in x, z, the contributions have at most one ψ'_m. Multiplying everything out, one finds the nonvanishing terms are,

$$\begin{aligned}\langle\langle \Psi^3 \rangle\rangle = 3 \lim_{N\to\infty} \Bigg[& \sum_{m=0}^{N} \langle\langle \Gamma \psi'_m \psi_N \rangle\rangle + \sum_{m=0}^{N} \langle\langle \Gamma \psi_N \psi'_m \rangle\rangle \\ & - \langle\langle \Gamma \psi_N^2 \rangle\rangle - \frac{1}{2}\langle\langle \Gamma \psi'_N \psi_N \rangle\rangle - \frac{1}{2}\langle\langle \Gamma \psi_N \psi'_N \rangle\rangle \Bigg]\end{aligned} \tag{3.12}$$

Calculating the inner product,

$$\begin{aligned}\langle\langle \Gamma \psi_m \psi_n \rangle\rangle &= \langle\langle \Omega^{m+1} Bc\Omega c \Omega^{n+1} \gamma^2 \rangle\rangle \\ &= \frac{m+n+3}{2\pi^2}\end{aligned} \tag{3.13}$$

we find,

$$\begin{aligned}\langle\langle \Psi^3 \rangle\rangle &= 3 \lim_{N\to\infty} \left[2\sum_{n=0}^{N} \frac{1}{2\pi^2} - \frac{2N+3}{2\pi^2} - \frac{1}{2\pi^2} \right] \\ &= 3 \lim_{N\to\infty} \left[\frac{2N+2}{2\pi^2} - \frac{2N+3}{2\pi^2} - \frac{1}{2\pi^2} \right] \\ &= -\frac{3}{\pi^2}\end{aligned} \tag{3.14}$$

and we have already calculated,

$$\langle\langle \Psi, Q_B \Psi \rangle\rangle = 6 \cdot \frac{1}{2\pi^2} = \frac{3}{\pi^2} \tag{3.15}$$

proving the equations of motion are satisfied.

One interesting feature of these proofs is that they work even at finite N, that is, the limit $N \to \infty$ was not necessary. Of course, at finite N we do not really have a solution, but this can be fixed up by replacing our regulated expression eq. (2.21) with,

$$\frac{K}{1-F^2} = \sum_{n=0}^{N} K\Omega^n - \left(\sum_{k=0}^{\infty} \frac{B_k}{k!} K^k + K \right) \Omega^N \tag{3.16}$$

For finite N substituting the Bernoulli power series is somewhat formal, but interestingly for $N = 0$ we recover the solution written in the $\mathcal{L}_0$ level expansion:[8]

$$
\begin{aligned}
\Psi &= -\sum_{n=0}^{\infty} \frac{B_n}{n!} FcK^n BcF - FB\gamma^2 F \\
&= -\sum_{n=0}^{\infty} \frac{B_n}{n!} \frac{d^n}{d\alpha^n}\bigg|_0 \psi_\alpha - \Gamma
\end{aligned}
\tag{3.17}
$$

Therefore, the fact that our calculation works independent of N implies that we have indirectly proven the energy in the $\mathcal{L}_0$ level expansion as well. If we like, we can repeat the proof in the new notation:

$$
\begin{aligned}
E &= -\frac{1}{6}\langle\langle \Psi, Q_B\Psi\rangle\rangle \\
&= -\frac{1}{6}\sum_{m,n=0}^{\infty} \frac{B_m B_n}{m!n!} \frac{\partial^m}{\partial\alpha^m}\bigg|_0 \frac{\partial^n}{\partial\beta^n}\bigg|_0 \langle\langle \psi_\alpha, Q_B\psi_\beta\rangle\rangle - \frac{1}{3}\sum_{m=0}^{\infty}\frac{B_m}{m!}\frac{\partial^m}{\partial\alpha^m}\bigg|_0 \langle\langle \psi_\alpha, Q_B\Gamma\rangle\rangle \\
&= -\frac{1}{6}\left[\left(\frac{B_0}{0!}\right)^2 \langle\langle \psi_0, Q_B\psi_0\rangle\rangle + 2\frac{B_0B_1}{0!1!}\langle\langle \psi_0', Q_B\psi_0\rangle\rangle\right] - \frac{1}{3}\frac{B_0}{0!}\langle\langle \psi_0, \Gamma\rangle\rangle
\end{aligned}
\tag{3.18}
$$

where in the last step we used the fact that second and higher derivatives of the inner products vanish. Plugging in eq. (3.7) and $B_1 = -\frac{1}{2}$,

$$
E = -\frac{1}{6}\left(\frac{2}{\pi^2} - \frac{1}{\pi^2}\right) - \frac{1}{3\pi^2} = -\frac{1}{2\pi^2} \tag{3.19}
$$

For the bosonic string, evaluating the energy in the $\mathcal{L}_0$ level expansion gives a very complicated asymptotic series, though the series can be resumed numerically to give a good approximation to the brane tension [1]. One advantage of this derivation is that we do not need to regulate the solution or worry about subtracting the correct ψ_N piece; these subtleties are implicitly taken care of in the $\mathcal{L}_0$ level expansion.

4. Conclusion

In this paper we have given a remarkably simple proof of Sen's conjectures in cubic superstring field theory. From an analytic perspective the solution appears to be as regular as Schnabl's solution for the bosonic string. From the perspective of the level expansion the situation is unclear. Given the Siegel gauge results (see table 1) we expect convergence to be irregular,[9] but perhaps the situation will improve at sufficiently high level.

[8] In the current notation, a state $F\phi F$ has $\mathcal{L}_0$ eigenvalue h if the operator insertion corresponding to the field ϕ has scaling dimension h in the cylinder coordinate system. K, B, c, γ^2 have dimension $1, 1, -1, -1$ respectively. Hence, for example, FcK^nBcF has $\mathcal{L}_0$ eigenvalues $n-1$ and $FB\gamma^2F$ has $\mathcal{L}_0$ eigenvalue 0.

[9] In fact, Ohmori [9] searched for, but failed to find a GSO(+) vacuum out to level (2,6). We find this worrisome, but it may be hard to identify a vacuum in level truncation because the cubic coupling $\langle\langle c, c, c\rangle\rangle$ vanishes by ϕ-momentum conservation. Therefore the auxiliary tachyon must depend on interactions with higher level fields to generate a minimum.

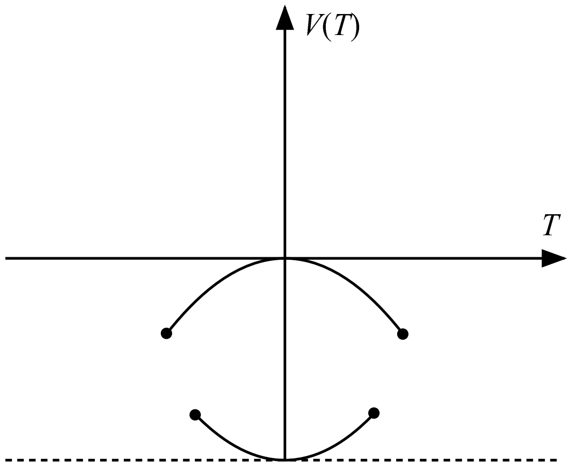

Figure 2: Conjectured form of the tachyon potential in Schnabl gauge for the cubic superstring. The minimum on the lower branch represents our analytic solution eq. (2.7). The dots at the edge of the curves represent points on the tachyon potential where Schnabl gauge breaks down.

Given the intrinsic uncertainties of the cubic theory, it is highly desirable to construct an analytic vacuum solution to Berkovits's WZW-type superstring field theory. Following the philosophy of refs. [22, 16], it is not difficult to construct formal solutions once an appropriate solution for the cubic equations of motion is known — in fact, one such vacuum solution has already been proposed in ref. [23]. However, the GSO(+) Berkovits solutions we have found seem to be singular in the $\mathcal{L}_0$ level expansion. We suspect that an analytic vacuum solution in the Berkovits theory will have to involve the GSO(−) sector in some nontrivial way. This may be expected from level truncation analysis, which shows a smooth double-well potential with minima for the tachyon at finite expectation value.

The biggest puzzle presented by our solution is the absence of any component in the GSO(−) sector. This brings up three apparent paradoxes:

1. There is an expectation, which so far has been unchallenged, that open string field theory on a particular brane system only describes classical solutions which are accessible via tachyon condensation. A BPS brane carries a conserved topological charge, so there is no means for it to decay to the vacuum, by tachyon condensation or otherwise. Thus it appears that the cubic superstring has "too many" solutions.

2. The intuitive picture of tachyon condensation suggests that the tachyon should roll off the top of the potential and come to rest at the vacuum with finite expectation value. However, the story here must be different. It seems possible that the tachyon potential in Schnabl gauge hits a singularity before a stable minimum is reached, and the GSO(+) solution lies on a disconnected branch directly below the unstable maximum (see figure 2). This scenario is made a more plausible from the level $\frac{5}{2}$ results of Raeymaekers [6]. It is also circumstantially supported by the (somewhat mysterious) late time rolling tachyon limit of Ellwood [25]. For the bosonic string, there seems

to be a sense in which the late time behavior of the rolling tachyon solution [14, 15] approaches Schnabl's solution. However, a similar limit for the superstring [16, 26] fails to yield a well-defined expression, suggesting a vacuum solution in Schnabl gauge with nonvanishing GSO($-$) sector may not exist.[10]

3. The third paradox comes from supersymmetry. Since the perturbative vacuum on the BPS brane has unbroken supersymmetry, one would not expect to find a state in the theory with lower energy.

It would be very interesting to gain concrete insight into these puzzles.

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A. Correlator

In this appendix we derive the correlator eq. (3.6) used to derive the inner products and energy in section 3. We start with:

$$\langle\langle \Omega^x B c \Omega^y c \Omega^z \gamma^2 \rangle\rangle = \left\langle Y_{-2} \int_{i\infty}^{-i\infty} \frac{dw}{2\pi i} b(w) c(z_1) c(z_2) \gamma^2(0) \right\rangle_{C_L}$$
$$L = x + y + z \qquad z_1 = y + z \qquad z_2 = z \tag{A.1}$$

as shown in figure 1. To simplify the b ghost insertion, we use the trick of Okawa [11]. We introduce a linear function on the cylinder $(z)_\delta$ with a branch cut at $\mathrm{Re}(z) = \delta$, and write the b insertion as a contour integral around this branch cut:

$$\int_{i\infty}^{-i\infty} \frac{dw}{2\pi i} b(z) = \frac{1}{L} \oint_{\mathrm{Re}(z)=\delta} \frac{dw}{2\pi i} (w)_\delta b(w) \tag{A.2}$$

The factor of $1/L$ is necessary because the discontinuity has height L for a cylinder of circumference L. We then deform the contour away from the branch cut to encircle the c insertions inside the correlator:

$$\begin{aligned}\langle\langle \Omega^x B c \Omega^y c \Omega^z \gamma^2 \rangle\rangle &= -\frac{1}{L} \left\langle Y_{-2} \left(\oint_{\mathrm{cs}} \frac{dw}{2\pi i} w b(w) c(z_1) c(z_2) \right) \gamma^2(0) \right\rangle_{C_L} \\ &= -\frac{z_1}{L} \langle Y_{-2} c(z_2) \gamma^2(0) \rangle_{C_L} + \frac{z_2}{L} \langle Y_{-2} c(z_1) \gamma^2(0) \rangle_{C_L}\end{aligned} \tag{A.3}$$

The remaining correlator can be evaluated by mapping back to the upper half plane and performing the necessary contractions. The answer is,

$$\langle Y_{-2} c(z) \gamma^2(0) \rangle_{C_L} = -\frac{L^2}{2\pi^2} \tag{A.4}$$

[10] For an interesting and different approach to marginal deformations, see refs. [27, 28]

The simplicity of this result is responsible for all of the drastic simplifications of the energy calculation. We can see that the basic structure is correct by inspection: The factor of L^2 is necessary because the insertions have total conformal dimension -2. Furthermore, the result must be independent of z because $c(z)$ and $\gamma^2(0)$ only have contractions with the picture changing operators at $\pm i\infty$, and by cylindrical symmetry these contractions are independent of the absolute or relative positions of these operators. Thus,

$$\begin{aligned}\langle\langle \Omega^x Bc\Omega^y c\Omega^z \gamma^2\rangle\rangle &= \frac{L}{2\pi^2}(z_1 - z_2)\\ &= \frac{x+y+z}{2\pi^2}y \end{aligned} \tag{A.5}$$

reproducing eq. (3.6).

References

[1] M. Schnabl, *Analytic solution for tachyon condensation in open string field theory*, *Adv. Theor. Math. Phys.* **10** (2006) 433 [hep-th/0511286].

[2] A. Sen, *Universality of the tachyon potential*, *JHEP* **12** (1999) 027 [hep-th/9911116].

[3] N. Berkovits, *SuperPoincaré invariant superstring field theory*, *Nucl. Phys.* **B 450** (1995) 90 [hep-th/9503099]; *A new approach to superstring field theory*, in the proceedings to the $32^{\rm nd}$ *International symposium Ahrenshoop on the theory of elementary particles*, *Fortschr. Phys.* **48** (2000) 31 [hep-th/9912121].

[4] N. Berkovits, *The tachyon potential in open Neveu-Schwarz string field theory*, *JHEP* **04** (2000) 022 [hep-th/0001084];
N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [hep-th/0002211];
P.-J. De Smet, *Tachyon condensation: calculations in string field theory*, hep-th/0109182.

[5] C.R. Preitschopf, C.B. Thorn and S.A. Yost, *Superstring field theory*, *Nucl. Phys.* **B 337** (1990) 363;
I.Y. Arefeva, P.B. Medvedev and A.P. Zubarev, *New representation for string field solves the consistence problem for open superstring field*, *Nucl. Phys.* **B 341** (1990) 464.

[6] J. Raeymaekers, *Tachyon condensation in string field theory: tachyon potential in the conformal field theory approach*, PhD thesis, KULeuven (2001).

[7] K. Ohmori, *A review on tachyon condensation in open string field theories*, hep-th/0102085.

[8] N. Berkovits, *Review of open superstring field theory*, hep-th/0105230; *The Ramond sector of open superstring field theory*, *JHEP* **11** (2001) 047 [hep-th/0109100].

[9] K. Ohmori, *Level-expansion analysis in NS superstring field theory revisited*, hep-th/0305103.

[10] I.Y. Aref'eva, A.S. Koshelev, D.M. Belov and P.B. Medvedev, *Tachyon condensation in cubic superstring field theory*, *Nucl. Phys.* **B 638** (2002) 3 [hep-th/0011117].

[11] Y. Okawa, *Comments on Schnabl's analytic solution for tachyon condensation in Witten's open string field theory*, *JHEP* **04** (2006) 055 [hep-th/0603159].

[12] T. Erler, *Split string formalism and the closed string vacuum*, *JHEP* **05** (2007) 083 [hep-th/0611200].

[13] T. Erler, *Split string formalism and the closed string vacuum. II*, *JHEP* **05** (2007) 084 [hep-th/0612050].

[14] M. Schnabl, *Comments on marginal deformations in open string field theory*, *Phys. Lett.* **B 654** (2007) 194 [hep-th/0701248].

[15] M. Kiermaier, Y. Okawa, L. Rastelli and B. Zwiebach, *Analytic solutions for marginal deformations in open string field theory*, hep-th/0701249.

[16] T. Erler, *Marginal solutions for the superstring*, *JHEP* **07** (2007) 050 [arXiv:0704.0930].

[17] I. Kishimoto and Y. Michishita, *Comments on solutions for nonsingular currents in open string field theories*, arXiv:0706.0409.

[18] I. Ellwood and M. Schnabl, *Proof of vanishing cohomology at the tachyon vacuum*, *JHEP* **02** (2007) 096 [hep-th/0606142].

[19] C. Imbimbo, *The spectrum of open string field theory at the stable tachyon vacuum*, hep-th/0611343.

[20] L. Rastelli, A. Sen and B. Zwiebach, *String field theory around the tachyon vacuum*, *Adv. Theor. Math. Phys.* **5** (2002) 353 [hep-th/0012251];
K. Okuyama, *Siegel gauge in vacuum string field theory*, *JHEP* **01** (2002) 043 [hep-th/0111087].

[21] L. Rastelli and B. Zwiebach, *Solving open string field theory with special projectors*, hep-th/0606131;
Y. Okawa, L. Rastelli and B. Zwiebach, *Analytic solutions for tachyon condensation with general projectors*, hep-th/0611110.

[22] K. Ohmori, *Comments on solutions of vacuum superstring field theory*, *JHEP* **04** (2002) 059 [hep-th/0204138]; *On ghost structure of vacuum superstring field theory*, *Nucl. Phys.* **B 648** (2003) 94 [hep-th/0208009].

[23] E. Fuchs and M. Kroyter, *Marginal deformation for the photon in superstring field theory*, arXiv:0706.0171.

[24] E. Fuchs and M. Kroyter, *On the validity of the solution of string field theory*, *JHEP* **05** (2006) 006 [hep-th/0603195].

[25] I. Ellwood, *Rolling to the tachyon vacuum in string field theory*, arXiv:0705.0013.

[26] Y. Okawa, *Analytic solutions for marginal deformations in open superstring field theory*, *JHEP* **09** (2007) 084 [arXiv:0704.0936]; *Real analytic solutions for marginal deformations in open superstring field theory*, *JHEP* **09** (2007) 082 [arXiv:0704.3612].

[27] E. Fuchs, M. Kroyter and R. Potting, *Marginal deformations in string field theory*, *JHEP* **09** (2007) 101 [arXiv:0704.2222].

[28] M. Kiermaier and Y. Okawa, *Exact marginality in open string field theory: a general framework*, arXiv:0707.4472.

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